A semiclassical approach to η/s bound through holography

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We consider the holographic principle, in its lightsheet formulation, in the semiclassical context of statistical-mechanical systems in classical Einstein spacetimes. A local condition, in terms of entropy and energy local densities of the material medium under consideration, is discussed, which turns out to be necessary and sufficient for the validity of the closely-related generalized covariant entropy bound. This condition is apparently a general consequence or expression of flat-spacetime quantum mechanics alone, without any reference to gravity. Using it, a lower bound $\eta/s \geq 1/4\pi$ can be derived, with the limit attained (in certain circumstances) by systems hydrodynamically dominated by radiation quanta.

For thermal theories with holographic gravitational dual, certain quantities, albeit unrelated to gravity, are however easily evaluated through string calculations in the equivalent dual gravitational theory. As far as the string calculations are performed in the semiclassical limit, they could be expected to be readable also in a conventional quantum approach, without reference to strings, for thermal systems at circumstances (to be defined) embodying the onset of the holographic duality above. The Kovtun-Son-Starinets (KSS) bound,[1] could be precisely an example of this. It says that for a large class of thermal field theories, even widely different from each other but always with gravity duals, $\eta/s = 1/4\pi$ (in Planck units, the units we use in this note), being also conjectured this to be in general a lower bound, at least for relativistic systems.

Now, holography is summarized, in the semiclassical context of material media living in the continuous Einstein spacetime, by the so-called generalized covariant entropy bound[2] (genB). This bound turns out to be universally true *iff* a lower limit is definitely put to the scale l of the statistical-mechanical description for the assigned local conditions, with this limit being apparently anyway required by the intrinsic space (and time[3]) quantum uncertainty of the constituent particles:[4,5]

$$l \geq \lambda \geq \frac{1}{\pi} \frac{s}{\rho + p} \equiv l^*, \tag{1}$$

where s, ρ, p are local entropy and energy density and pressure respectively and λ is the typical wavelength of the constituent particles. The genB can be saturated

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(for peculiar geometric configurations) by systems which have $\lambda = l^*$, i.e. the most entropic ones (typically, ultrarelativistic gases).[5]

Exploiting the notion of l^* we have just introduced we can calculate η/s (along the lines of Ref. 6). For nonrelativistic systems, for example a gas at ordinary conditions, we have $\eta = \frac{1}{3}L\rho a$ with L the correlation length and a the thermal velocity of the constituent particles, so that

$$\frac{\eta}{s} = \frac{1}{3}\frac{L\rho a}{s} \simeq \frac{1}{3}L\frac{\rho + p}{s}a = \frac{1}{3\pi}\frac{L}{l^*}a, \tag{2}$$

where use is made of the expression (1) for l^*.

Now, if ρ, s, p are allowed to increase while $\frac{\rho+p}{s}$ and λ are held fixed (for a gas this means to increase number density with intensive parameters fixed), L decreases, but without going below its quantum mechanical limit λ. If we assume that this limiting condition can actually be reached we have

$$\left(\frac{\eta}{s}\right)_{min} = \frac{1}{3\pi}\frac{\lambda}{l^*}a. \tag{3}$$

Here a can be also very near to 0 so that the KSS bound could be violated. However, considering explicitly the case of a Boltzmann gas we see this is not the case. We have in fact for it[5] $\frac{\lambda}{l^*} \gg 1$ with

$$\frac{\lambda}{l^*} = (2\pi)^{3/2}\sqrt{\frac{m}{T}}\;\frac{1}{\chi + \frac{f+2}{2}} \propto \sqrt{\frac{m}{T}} \propto \frac{1}{a}, \tag{4}$$

so that in $(\eta/s)_{min}$ no dependence on a is left. In this expression m is the mass of constituent particles, χ is a number $\simeq 0$ when conditions are such that $L \simeq \lambda$, f is the number of degrees of freedom per particle, and use is made of the relation between temperature and thermal velocity, $T = \frac{1}{3}ma^2$. Putting numbers in, if we assume $f = 2$ we find $(\eta/s)_{min} \approx 0.7$. Thus the KSS bound seems can be satisfied also for nonrelativistic systems, no matter how small a can be.

Let us consider ultrarelativistic systems consisting of interacting radiation. We can have in mind massive particles with statistical equilibrium determined by collisions with radiation quanta (the particles only act as mechanism to transfer momentum through the radiation field) or directly a gas of photons and ultrarelativistic electrons and positrons, or gluons interacting among themselves and with ultrarelativistic quarks. Assuming $\eta \approx \frac{1}{3}\tau\rho_\gamma$,[7] where τ is the average time for a quantum to collide and ρ_γ is the energy density of radiation, we get

$$\frac{\eta}{s_\gamma} \approx \frac{1}{3}\tau\frac{\rho_\gamma}{s_\gamma} = \frac{1}{4}\tau\frac{\rho_\gamma + p_\gamma}{s_\gamma} = \frac{1}{4\pi}\frac{\tau}{l^*_\gamma} = \frac{1}{4\pi}\frac{L}{l^*_\gamma}, \tag{5}$$

with L the average distance for a quantum to collide and s_γ the entropy density of radiation. Now, for a gas of radiation quanta still L cannot be smaller than its

quantum limit λ_γ. If we assume that, for assigned T, L can decrease down to the limit $L \to \lambda_\gamma = L_{min}$ (which, actually, amounts to imply strong coupling, since, at the limiting conditions of one potential collision in every λ_γ, $L = \frac{1}{\sigma n} \approx \frac{\lambda_\gamma}{g^2}$, where n is number density ($= \lambda_\gamma^{-3}$ for radiation) and for σ the expression of Thompson cross section is used in terms of the coupling constant g), we get

$$\left(\frac{\eta}{s}\right)_{min} \approx \frac{1}{4\pi}\frac{\lambda_\gamma}{l^*_\gamma} = \frac{1}{4\pi}, \tag{6}$$

where last equality comes from the fact that for radiation $\lambda = l^*$, from what mentioned above.

Apparently, the KSS limit must be there, and the conditions for it to be attained are the fluid has $\lambda = l^*$ (an "holographic" fluid, i.e. able to attain the generalized covariant entropy bound) and $L = \lambda$. We should thus define the "holographic duality conditions" for our semiclassic fluids to be $\lambda = l^* = L$.

As far as the QCD matter produced at RHIC can be described in terms of a fluid of strongly coupled radiation quanta, the very low values of η/s found at RHIC[8] could thus find an explanation in what we have seen.

In conclusion, to the value $\eta/s = 1/4\pi$ it is possible to arrive both through string theoretical calculations in the gravity dual of holographic thermal theories and, we have seen, through ordinary flat-spacetime quantum arguments for fluids at "holographic duality" $\lambda = l^* = L$.

In Ref. 9 a different non-string-theoretical argument bringing to the KSS limit is presented, still relying on l^* concept.

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